Rules:

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| --- | --- |
| f(x)=f(0)+f^'(0)x+(f^('')(0))/(2!)x^2+(f^((3))(0))/(3!)x^3+...+(f^((n))(0))/(n!)x^n+.... | (1) |

Maclaurin series are named after the Scottish mathematician Colin Maclaurin.

The Maclaurin series of a function f(x) up to order n may be found using [Series](http://reference.wolfram.com/language/ref/Series.html)[*f*, {*x*, 0, *n*}]. The nth term of a Maclaurin series of a function f can be computed in the [Wolfram Language](http://www.wolfram.com/language/) using [SeriesCoefficient](http://reference.wolfram.com/language/ref/SeriesCoefficient.html)[*f*, {*x*, 0, *n*}] and is given by the inverse [Z-transform](http://mathworld.wolfram.com/Z-Transform.html)

|  |  |
| --- | --- |
| a_n=Z^(-1)[1/x](n). | (2) |

Maclaurin series are a type of [series expansion](http://mathworld.wolfram.com/SeriesExpansion.html) in which all terms are nonnegative integer powers of the variable. Other more general types of series include the [Laurent series](http://mathworld.wolfram.com/LaurentSeries.html) and the [Puiseux series](http://mathworld.wolfram.com/PuiseuxSeries.html).

Maclaurin series for common functions include

|  |  |
| --- | --- |
| 1/(1-x)=1+x+x^2+x^3+x^4+x^5+... | (3) |
| for -1<x<1 | (4) |
| cn(x,k)=1-1/2x^2+1/(24)(1+4k^2)x^4+... | (5) |
| cosx=1-1/2x^2+1/(24)x^4-1/(720)x^6+... | (6) |
| for -infty<x<infty | (7) |
| cos^(-1)x=1/2pi-x-1/6x^3-3/(40)x^5-5/(112)x^7-... | (8) |
| for -1<x<1 | (9) |
| coshx=1+1/2x^2+1/(24)x^4+1/(720)x^6+1/(40,320)x^8+... | (10) |
| cot^(-1)x=1/2pi-x+1/3x^3-1/5x^5+1/7x^7-1/9x^9+... | (11) |
| dn(x,k)=1-1/2k^2x^2+1/(24)k^2(4+k^2)x^4+... | (12) |
| erf(x)=1/(sqrt(pi))(2x-2/3x^3+1/5x^5-1/(21)x^7+...) | (13) |
| e^x=1+x+1/2x^2+1/6x^3+1/(24)x^4+... | (14) |
| for -infty<x<infty | (15) |
| _2F_1(alpha,beta;gamma;x)=1+(alphabeta)/(1!gamma)x+(alpha(alpha+1)beta(beta+1))/(2!gamma(gamma+1))x^2+... | (16) |
| ln(1+x)=x-1/2x^2+1/3x^3-1/4x^4+... | (17) |
| for -1<x<=1 | (18) |
| ln((1+x)/(1-x))=2x+2/3x^3+2/5x^5+2/7x^7+... | (19) |
| for -1<x<1 | (20) |
| secx=1+1/2x^2+5/(24)x^4+(61)/(720)x^6+(277)/(8064)x^8+... | (21) |
| sechx=1-1/2x^2+5/(24)x^4-(61)/(720)x^6+(277)/(8064)x^8+... | (22) |
| sinx=x-1/6x^3+1/(120)x^5-1/(5040)x^7+... | (23) |
| for -infty<x<infty | (24) |
| sin^(-1)x=x+1/6x^3+3/(40)x^5+5/(112)x^7+(35)/(1152)x^9+... | (25) |
| sinhx=x+1/6x^3+1/(120)x^5+1/(5040)x^7+1/(362880)x^9+... | (26) |
| sinh^(-1)x=x-1/6x^3+3/(40)x^5-5/(112)x^7+(35)/(1152)x^9-... | (27) |
| sn(x,k)=x-1/6(1+k^2)x^3+1/(120)(1+14k^2+k^4)x^5+... | (28) |
| tanx=x+1/3x^3+2/(15)x^5+(17)/(315)x^7+(62)/(2835)x^9+... | (29) |
| tan^(-1)x=x-1/3x^3+1/5x^5-1/7x^7+... | (30) |
| for -1<x<1 | (31) |
| tanhx=x-1/3x^3+2/(15)x^5-(17)/(315)x^7+(62)/(2835)x^9+... | (32) |
| tanh^(-1)x=x+1/3x^3+1/5x^5+1/7x^7+1/9x^9+.... | (33) |

The explicit forms for some of these are

|  |  |
| --- | --- |
| 1/(1-x)=sum_(n=0)^(infty)x^n | (34) |
| cosx=sum_(n=0)^(infty)((-1)^n)/((2n)!)x^(2n) | (35) |
| cos^(-1)x=pi/2-sum_(n=0)^(infty)(Gamma(n+1/2))/(sqrt(pi)(2n+1)n!)x^(2n+1) | (36) |
| coshx=sum_(n=0)^(infty)1/((2n)!)x^(2n) | (37) |
| cot^(-1)x=pi/2-sum_(n=0)^(infty)((-1)^n)/(2n+1)x^(2n+1) | (38) |
| e^x=sum_(n=0)^(infty)1/(n!)x^n | (39) |
| erf(x)=sum_(n=0)^(infty)(2(-1)^n)/(sqrt(pi)(2n+1)n!)x^(2n+1) | (40) |
| _2F_1(alpha,beta;gamma,x)=sum_(n=0)^(infty)((alpha)_n(beta)_n)/((gamma)_n)(x^n)/(n!) | (41) |
| ln(1+x)=sum_(n=1)^(infty)((-1)^(n+1))/nx^n | (42) |
| ln((1+x)/(1-x))=sum_(n=1)^(infty)2/((2n-1))x^(2n-1) | (43) |
| secx=sum_(n=0)^(infty)((-1)^nE_(2n))/((2n)!)x^(2n) | (44) |
| sechx=sum_(n=0)^(infty)(E_(2n))/((2n)!)x^(2n) | (45) |
| sinx=sum_(n=0)^(infty)((-1)^n)/((2n+1)!)x^(2n+1) | (46) |
| sin^(-1)x=sum_(n=0)^(infty)(Gamma(n+1/2))/(sqrt(pi)(2n+1)n!)x^(2n+1) | (47) |
| sinhx=sum_(n=0)^(infty)1/((2n+1)!)x^(2n+1) | (48) |
| sinh^(-1)x=sum_(n=0)^(infty)(P_(2n)(0))/(2n+1)x^(2n+1) | (49) |
| tanx=sum_(n=0)^(infty)((-1)^n2^(2n+2)(2^(2n+2)-1)B_(2n+2))/((2n+2)!)x^(2n+1) | (50) |
| tan^(-1)x=sum_(n=1)^(infty)((-1)^(n+1))/(2n-1)x^(2n-1) | (51) |
| tanhx=sum_(n=1)^(infty)(2^(2n)(2^(2n)-1)B_(2n))/((2n)!)x^(2n-1) | (52) |
| tanh^(-1)x=sum_(n=1)^(infty)1/(2n-1)x^(2n-1), | (53) |